

WEIGHTED HARDY–TYPE INEQUALITIES FOR MONOTONE CONVEX FUNCTIONS WITH SOME APPLICATIONS

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Abstract. In this paper, we establish some new refined weighted Hardy-type inequalities involving monotone convex functions. We give the results for some special kernels of Riemann-Liouville and Weyl's operators as applications. Also we discuss some related dual cases. At the end, we prove some refined G. H. Hardy-type inequalities for different kinds of fractional integrals and fractional derivatives.

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