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WELL–POSEDNESS AND REGULARITY OF THE CAUCHY PROBLEM FOR NONLINEAR FRACTIONAL IN TIME AND SPACE EQUATIONS

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Abstract. The purpose is to study the Cauchy problem for non-linear in time and space pseudo-differential equations. These include the fractional in time versions of Hamilton-Jacobi-Bellman (HJB) equations governing the limits of controlled scaled Continuous Time Random Walks (CTRWs). As a preliminary step which is of independent interest we analyse the corresponding linear equation proving its well-posedness and smoothing properties.

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