

HERMITE–HADAMARD TYPE INEQUALITIES FOR RIEMANN–LIOUVILLE FRACTIONAL INTEGRALS OF (α, m) –CONVEX FUNCTIONS

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Abstract. In the paper, the authors establish some new Hermite-Hadamard type inequalities for Riemann-Liouville fractional integrals of functions whose derivatives in absolute value are of (α, m) -convexity.

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