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EXISTENCE OF MILD SOLUTIONS FOR NONLOCAL SEMILINEAR FRACTIONAL EVOLUTION EQUATIONS

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Abstract. In this paper, we investigate a class of semilinear fractional evolution equations with nonlocal initial conditions given by

$$\begin{cases} \dfrac{\mathrm{d}^q u(t)}{\mathrm{d}t^q} = Au(t) + (Fu)(t), \ \ t \in I, \\ u(0) + g(u) = u_0, \end{cases} \tag{1}$$

where $0 < q < 1$, I is a compact interval. Sufficient conditions for the existence of mild solutions for the equation (1) are derived. The main tools include Laplace transform, Arzela-Ascoli's Theorem, Schauder's fixed point theorem and Operator theorem.

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