

## POSITIVE SOLUTIONS OF AN INTEGRODIFFERENTIAL MULTI-POINT INITIAL VALUE PROBLEM WITH FRACTIONAL ORDER

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*Abstract.* The aim of this paper is to obtain existence results for a nonlinear integrodifferential multi-point initial value problem of fractional order by using fractional calculus and fixed point theory. Then, we study the positivity of the obtained solution.

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