

ON A LOCAL SOLVABILITY OF THE MULTIDIMENSIONAL MUSKAT PROBLEM WITH A FRACTIONAL DERIVATIVE IN TIME ON THE BOUNDARY CONDITION

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Abstract. In this paper, we analyze anomalous diffusion version of the multidimensional Muskat problem without surface tension on a free boundary. We prove the existence and uniqueness of the classical solution to this moving boundary problem locally in time.

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