

FINITE DIFFERENCE METHOD FOR SOLVING THE SPACE–TIME FRACTIONAL WAVE EQUATION IN THE CAPUTO FORM

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Abstract. In this paper a space-time fractional wave equation on a finite domain is considered. The time and space fractional derivative are described in the Caputo sense. We propose a finite difference scheme to solve the space-time fractional wave equation. We discuss about stability and convergence of the method and prove that the finite difference scheme is unconditionally stable and convergent with $(\tau^2 + h)$, where τ and h are time and space steps respectively.

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