

## MAXIMAL SOLUTIONS TO FRACTIONAL DIFFERENTIAL EQUATIONS

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*Abstract.* When do fractional differential equations have maximal solutions? This note discusses this question in the following way. Firstly, a comparison theorem is formulated that involves fractional differential inequalities. Secondly, a sequence of approximative problems involving polynomials is analyzed and it is shown that there is a subsequence of solutions whose limit is the maximal solution to the original problem of interest. In particular, the interval of existence for the maximal solution is the optimal length, aligning with best practice in the local theory of existence of solutions. We achieve this through an application of the Arzela–Ascoli Theorem and our aforementioned comparison result. A YouTube video by the author designed to complement this work is available at <http://tinyurl.com/MaxFracDE>.

*Mathematics subject classification* (2010): 34A08, 34A12.

*Keywords and phrases:* Maximal solution, comparison theorem, existence of solutions, fractional differential equations, initial value problem.

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