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OPIAL–TYPE INEQUALITIES FOR FRACTIONAL INTEGRAL OPERATOR INVOLVING MITTAG—LEFFLER FUNCTION

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Abstract. In this paper we give generalization of Opial-type inequalities by using generalized fractional integral operator involving generalized Mittag–Leffler function. We deduce some results which already have been proved. Also we consider n-exponential convexity of some non-negative differences of inequalities involving Mittag-Leffler function and deduce their exponential convexity and log-convexity.

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