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LIE SYMMETRY ANALYSIS TO GENERAL TIME–FRACTIONAL KORTEWEG–DE VRIES EQUATIONS

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Abstract. In present paper, two class of the general time-fractional Korteweg-de Vries equations (KdVs) are considered, a systematic investigation to derive Lie point symmetries of the equations are presented and compared. Each of them has been transformed into a nonlinear ordinary differential equation with a new independent variable are investigated. The derivative corresponding to time-fractional in the reduced formula is known as the Erdélyi-Kober fractional derivative.

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