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EXISTENCE AND MULTIPLICITY RESULTS FOR THE BOUNDARY VALUE PROBLEM OF NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we devote to investigation of the existence of positive solutions for the boundary value problem of nonlinear fractional differential equations

$$\begin{cases} D_{0^+}^{\alpha} u(t) + f(t, u(t)) = 0, \ \ 0 < t < 1, \\ u(0) = u'(0) = \cdots u^{(n-2)}(0) = D_{0^+}^{\beta} u(1), \end{cases}$$

where $D_{0^+}^{\alpha}$, $D_{0^+}^{\beta}$ are the standard Riemann-Liouville fractional derivative, $n-1 < \alpha \leqslant n$, $n-2 \leqslant \beta \leqslant n-1$, $n \geqslant 3$. By means of constructing an exact cone of the Banach space and fixed-point theorem, some new multiplicity results for the boundary value problem are obtained. The interest is that we establish the theorems of the existence of infinitely many positive solutions.

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REFERENCES

[1] S. G. SAMKO, A. A. KILBAS, O. I. MARICHEV, *Fractional Integral And Derivatives (Theory and Applications)*, Gordon and Breach, Switzerland, 1993.
[2] I. PODLUBNY, *Fractional Differential Equations*, in: Mathematics in Science and Engineering, vol. **198**, Academic Press, New Tork, London, Toronto, 1999.
[3] O. P. AGRAWAL, *Formulation of Euler-Lagrange equations for fractional variational problems*, J. Math. Anal. Appl. **272** (2002) 368–379.
[4] A. BABAKHANI, V. D. GEJJI, *Existence of positive solutions of nonlinear fractional differential equations*, J. Math. Anal. Appl. **278** (2003) 434–442.
[5] Z. B. BAI, H. S. LU, *Positive solutions for boundary value problem of nonlinear fractional differential equation*, J. Math. Anal. Appl. **311** (2005) 495–505.
[6] R. P. AGARWAL, M. BENCHOHRA, S. HAMANI, *Boundary value problem for fractional differential equations*, Adv. Stud. Contemp. Math. **16** (2) (2008) 181–196.
[7] A. A. KILBAS, H. M. SRIVASTAVA, J. J. TRUJILLO, *Theory and Applications of Fractional Differential Equations*, in: North-Holland Mathematics Studies, vol. 204, Elsevier Science B. V., Amsterdam, 2006.
[8] H. M. SRIVASTAVA, R. K. SAXENA, *Operators of fractional integration and their applications*, Appl. Math. Comput. **118** (2001) 1–52.
[9] B. AHMAD, S. SIVASUNDARAM, *Existence and uniqueness results for nonlinear boundary value problems of fractional differential equations with separated boundary conditions*, Commun. Appl. Anal. **13** (2009) 121–228.
[10] H. M. SRIVASTAVA, S. OWA, K. NISHIMOTO, *Some fractional differintegral equations*, J. Math. Anal. Appl. **106** (1985) 360–366.
[11] V. LAKSHMIKANTHAM, A. S. VATSALA, *General uniqueness and monotone iterative or fractional differential equations*, Appl. Math. Lett. **21** (2008) 828–834.

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[12] C. BAI, J. FANG, *The existence of a positive solution for a singular coupled system of nonlinear fractional dierential equations*, Appl. Math. Comput. **150** (2004) 611–621.
[13] V. LAKSHMIKANTHAM, A. S. VATSALA, *Basic theory of fractional dierential equations*, Nonlinear Anal. TMA. **69** (2008) 2677–2682.
[14] E. KAUFMANN, E. MBOUMI, *Positive solutions of a boundary value problem for a nonlinear fractional differential equation*, Electron. J. Qual. Theory Differ. Equ. **2008** (2008), No. 3, 1–11.
[15] M. A. KRASNOSELSKII, *Positive Solutions of Operator Equations*, Groningen, Netherlands, 1964.
[16] D. JIANG, C. YUAN, *The properties of the Green function for Dirichlet-type boundary value problem of a nonlinear fractional differential equations and its application*, Nonlinear Anal. **72** (2010) 710–719.
[17] C. F. LI, X. N. LUO, Y. ZHOU, *Existence of positive solution of the boundary value problem for nonlinear fractional differential equations*, Comput and Math Appl. **59** (2010) 1363–1375.
[18] S. H. LIANG, J. H. ZHANG, *Positive solutions for boundary value problem of nonlinear fractional differential equation*, Nonlinear Anal. **71** (2009) 5545–5550.