EXISTENCE AND NONEXISTENCE OF POSITIVE SOLUTIONS FOR A SYSTEM OF NONLINEAR SINGULAR FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we establish sufficient conditions for the existence and nonexistence of positive solutions to the following nonlinear fractional differential system

$$
\begin{cases}
D^\alpha u(t) + a(t)f(t,u,v) = 0 & \text{in } (0,1), \\
D^\beta v(t) + b(t)g(t,u,v) = 0 & \text{in } (0,1), \\
\quad u(0) = 0,\ u(1) = 0,\ u'(0) = 0, \\
\quad v(0) = 0,\ v(1) = 0,\ v'(0) = 0,
\end{cases}
\tag{P}
$$

where $2 < \alpha, \beta \leqslant 3$, $a,b \in \mathscr{C}((0,1),[0,+\infty))$ and the functions f,g belong to $\mathscr{C}([0,1] \times [0,+\infty) \times [0,+\infty),[0,+\infty))$ and satisfy some appropriate conditions. Our analysis relies on Krasnoselskii fixed point theorem. Some examples are given to illustrate our results.

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