

## THREE-POINT BOUNDARY VALUE PROBLEMS WITH DELTA RIEMANN-LIOUVILLE FRACTIONAL DERIVATIVE ON TIME SCALES

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**Abstract.** In this paper, we establish the criteria for the existence and uniqueness of solutions of a three-point boundary value problem for a class of fractional differential equations on time scales. By using some well known fixed point theorems, sufficient conditions for the existence of solutions are established. An illustrative example is also presented.

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