

## SOME NEW OSCILLATION CRITERIA FOR A CLASS OF NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS

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**Abstract.** Several oscillation criteria are established for nonlinear fractional differential equations of the form

$$\left\{ a(t) \left[ (r(t)g(D^\alpha x(t)))' \right]^\eta \right\}' - F \left( t, \int_t^\infty (v-t)^{-\alpha} x(v) dv \right) = 0,$$

where  $D^\alpha x(t)$  is the Liouville right-side fractional derivative of order  $\alpha \in (0, 1)$  of  $x(t)$ ,  $\eta = \frac{2n+1}{2m+1}$ , and  $n, m \in \mathbb{N}$ .  $F(t, G) \in C([t_0, \infty) \times \mathbb{R}; \mathbb{R})$ , and there exists function  $q(t) \in C^1([t_0, \infty); \mathbb{R}_+)$  such that  $\frac{F(t, G)}{G^\eta} \geq q(t)$  for  $G \neq 0$  and  $x \neq 0$ ,  $t \geq t_0$ . We also give some examples to illustrate the main results.

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