FRACTIONAL CALCULUS OF VARIATIONS WITH
A GENERALIZED FRACTIONAL DERIVATIVE

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Abstract. In this paper, we introduce a generalization of the Hilfer-Prabhakar derivative and obtain the Euler-Lagrange equations and Hamiltonian formulation with respect to this fractional derivative in the theory of fractional calculus of variations. Also, we get a sufficient condition for optimality.

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