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SOME PROPERTIES OF PRABHAKAR–TYPE FRACTIONAL CALCULUS OPERATORS

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Abstract. In this paper we study some properties of the Prabhakar integrals and derivatives and of some of their extensions such as the regularized Prabhakar derivative or the Hilfer–Prabhakar derivative. Some Opial- and Hardy-type inequalities are derived. In the last section we point out on some relationships with probability theory.

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Paper FDC-06-05

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