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ABOUT THE EXISTENCE OF SOLUTIONS FOR A HYBRID NONLINEAR GENERALIZED FRACTIONAL PANTOGRAPH EQUATION

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Abstract. The main purpose of this paper is to study the existence of solutions for the following hybrid nonlinear fractional pantograph equation

$$\begin{cases} D^{\alpha}_{0+}\left[\dfrac{x(t)}{f(t,x(t),x(\varphi(t)))}\right] = g(t,x(t),x(\rho(t))), \quad 0<t<1 \\ x(0)=0, \end{cases}$$

where $\alpha \in (0,1)$, φ and ρ are functions from $[0,1]$ into itself and D^{α}_{0+} denotes the Riemann-Liouville fractional derivative. The main tool of our study is a generalization of Darbo's fixed point theorem associated to measures of non-compactness. Also, we present an example illustrating our results.

Mathematics subject classification (2010): 45G10, 45M99, 47H09.
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REFERENCES

[1] A. A. KILBAS, H. M. SRIVASTAVA, J. J. TRUJILLO, *Theory and Applications of Fractional Differential Equations*, vol. **204**, North-Holland Mathematical Studies, Elsevier Science B. V., Amsterdam, 2006.
[2] I. PODLUBNY, *Fractional Differential Equations*, vol. **198**, Academic Press, New York, 1999.
[3] K. S. MILLER, B. ROSS, *An Introduction to the Fractional Calculus and Fractional Differential Equations*, John Wiley and Sons, New York, 1993.
[4] J. SABATIER, D. P. AGRAWAL, J. A. T. MACHADO, *Advances in Fractional Calculus: Theoretical Developments and Applications in Physics and Engenieering*, Springer, Dordrecht, 2007.
[5] J. R. OCKENDON, A. B. TAYLER, *The dynamics of a current collection system for an electric locomotive*, Proc. R.Soc. Lond. Ser. A **332** (1971), 447–468.
[6] M. BUHMANN, A. ISERLES, *Stability of the discretized pantograph differential equation*, Math. Comp. **60** (1993), 575–589.
[7] K. GUAN, J. SHEN, *On first-order neutral differential equations of Euler form with unbounded delays*, Appl. Math. Comput. **189** (2007), 1419–1427.
[8] A. ISERLES, *On the generalized pantograph functional-differential equation*, European J. Appl. Math. **4** (1993), 1–38.
[9] X. LIU, J. SHEN, *Asymptotic behavior of solutions of impusive neutral differential equations*, Appl. Math. Lett. **12** (1999), 51–58.
[10] M. SEZER, A. AKYÜZ-DASCIOĞLU, *A Taylor method for numerical solution of generalized pantograph equations with linear functional argument*, J.Comput. Appl. Math. **200** (2007), 217–225.
[11] K. BALACHANDRAN, V. KIRUTHIKA, J. J. TRUJILLO, *Existence of solutions of nonlinear fractional pantograph equations*, Acta Math.Sci. **33** (2013), 712–720.
[12] B. C. DHAGE, V. LAKSHMIKANTHAM, *Basic results on hybrid differential equations*, Nonlinear Anal. Hybrid Syst. **4** (2010), 414–424.

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[13] Y. ZHAO, S. SUN, Z. HAN, Q. LI, *Theory of fractional hybrid differential equations*, Comput. Math. Appl. **62** (2011), 1312–1324.
[14] M. A. DARWISH, K. SADARANGANI, *Existence of solutions for hybrid fractional pantograph equations*, App. Anal.Discrete Math. **9** (2015), 150–167.
[15] M. JLELI, M. MURSALEEN, B. SAMET, *q-integral equations of fractional orders*, Electron. J. Diff. Equ. **2016**, 17 (2016), 1–14.
[16] J. BANAS, K. GOEBEL, *Measures of Non-compactness in Banach Spaces*, Lecture Notes in Pure and Applied Mathematics **60**, Marcel Dekker, New York, 1980.
[17] G. DARBO, *Punti uniti in transformazioni a codominio noncompatto*, Rendiconti del Seminario Matematico della Universitá di Padova **24** (1955), 84–92.
[18] A. AGHAJANI, J. BANAS, N. SABZALI, *Some generalizations of Darbo fixed point theorem and applications*, Bulletin of the Belgian Mathematical Society Simon Stevin **20** (2013), 345–358.
[19] A. SAMADI, M. B. GHAEMI, *An extension of Darbo fixed point theorem and its applications to coupled fixed point and integral equations*, Filomat **28** (2014), 879–886.
[20] R. ARAB, *Some fixed point theorems in generalized Darbo fixed point theorem and the existence of solutions for system of integral equations*, J. Korean Math. Soc. **52** (2015), 125–139.
[21] M. JLELI, E. KARAPINAR, D. O'REGAN, B. SAMET, *Some generalizations of Darbo's theorem and applications to fractional integral equations*, Fixed Point Theory and Applications **2016**, 11 (2016).
[22] M. JLELI, B. SAMET, *A new generalization of the Banach contraction principle*, J. Inequal. Appl. **2014**, 38 (2014):8 pp.
[23] J. BANAS, L. OLSZOWY, *On a class of measures of noncompactness in Banach algebras and their application to nonlinear integral equations*, Z. Anal. Anwend. **28** (2009), 475–498.