

A PROBLEM WITH AN INTEGRAL BOUNDARY CONDITION FOR A TIME FRACTIONAL DIFFUSION EQUATION AND AN INVERSE PROBLEM

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Abstract. For a linear inhomogeneous time fractional diffusion equation on bounded cylindrical domain the problem with an integral boundary condition is studied. The inverse problem for the restoration of the whole right-hand side of the equation is also studied. The conditions of the solvability and the unique solvability of these problems are founded.

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