

TIME-FRACTIONAL DIFFUSION EQUATION WITH DYNAMICAL BOUNDARY CONDITION

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Abstract. We establish the unique solvability in Hölder spaces for an initial-boundary problem for fractional diffusion equation with fractional dynamic boundary condition.

Mathematics subject classification (2010): 35S15.

Keywords and phrases: Fractional derivative, Hölder class, classical solution.

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