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SOLVABILITY AND POSITIVE SOLUTIONS OF A SYSTEM OF HIGHER ORDER FRACTIONAL BOUNDARY VALUE PROBLEM WITH INTEGRAL CONDITIONS

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Abstract. The main purpose of this paper is to study the problem of the existence, uniqueness and positivity of solutions of a system of higher order fractional differential equations with boundary value problem expressed by fractional and integral conditions. Using fixed point theorems, we discuss the existence and the uniqueness of solutions of this problem, and we apply Guo-Krasnoselskii's fixed point theorem in cone to study the existence of positive solutions. We give some examples to illustrate our results.

Mathematics subject classification (2010): 26A33, 34B15, 34B18, 34B27.
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