Fractional Differential Calculus
Volume 6, Number 2 (2016), 201–217
doi:10.7153/fdc-06-13

THE NEHARI MANIFOLD FOR A SINGULAR ELLIPTIC EQUATION INVOLVING THE FRACTIONAL LAPLACE OPERATOR

ABDELJABBAR GHANMI AND KAMEL SAOUDI

Abstract. In this work we study the following singular problem involving the fractional Laplace operator:

$$(\mathrm{P}_\lambda)\begin{cases} \mathscr{L}u = \frac{a(x)}{u^\gamma} + \lambda f(x,u) & \text{in } \Omega; \\ u = 0, & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$, $N \geqslant 2$ be a bounded smooth domain, $a \in C(\Omega)$, λ is a positive parameter and $0 < \gamma < 1$, $2 < r < 2_s^*$ where $2_s^* = \frac{N2}{N-2s}$. Under appropriate assumptions on the function K and the function f and we employ the method of Nehari manifold in order to show the existence of $T_{r,\gamma}$ such that for all $\lambda \in (0, T_{r,\gamma})$, problem (P_λ) has at least two solutions.

Mathematics subject classification (2010): 34B15, 37C25, 35R20.
Keywords and phrases: Fractional Laplace operator, the method of Nehari manifold, singular elliptic equation, multiple positive solutions.

REFERENCES

[1] C. O. ALVES, A. EL HAMIDI, *Nehari manifold and existence of positive solutions to a class of quasilinear problems*, Nonlinear Anal. **60** (2005) 611–624.
[2] J. P. AUBIN, I. EKELAND, *Applied Nonlinear Analysis*, Pure Appl. Math., Wiley-Interscience Publications, 1984.
[3] D. APPLEBAUM, *Lévy Processes and Stochastic Calculus*, seconded., Camb. Stud. Adv. Math., **116**, Cambridge University Press, Cambridge, 2009.
[4] B. BARRIOS, E. COLORADO, A. DE PABLO, U. SANCHEZ, *On some critical problems for the fractional Laplacian operator*, J. Differential Equations **252** (2012), 6133–6162.
[5] K. J. BROWN, Y. ZHANG, *The Nehari manifold for a semilinear elliptic equation with a sign-changing weight function*, J. Differential Equations **193** (2003) 481–499.
[6] X. CABRÉ, J. TAN, *Positive solutions of nonlinear problems involving the square root of the Laplacian*, Adv. Math. **224** (2010), 2052–2093.
[7] L. CAFFARELLI, L. SILVESTRE, *An extension problem related to the fractional Laplacian*, Comm. Partial Differential Equations **32** (2007), 1245–1260.
[8] M. M. COCLITE AND G. PALMIERI, *On a singular nonlinear Dirichlet problem*, Comm. Partial Differential Equations **14** (1989), no. 10, 1315–1327.
[9] H. CHEN, H. HAJAIEJ, Y. WANG, *On a class of semilinear fractional elliptic equations involving outside Dirac data*, Nonlinear Anal. **125** (2015), 639–658.
[10] M. G. CRANDALL, P. H. RABINOWITZ AND L. TARTAR, *On a Dirichlet problem with a singular nonlinearity*, Comm. Partial Differential Equations **2** (1977), no. 2, 193–222.
[11] R. CONT AND P. TANKOV, *Financial Modelling with Jump Processes*, Chapman and Hall/CRC Financ. Math. Ser. 2004.
[12] P. DRABEK AND S. I. POHOZAEV, *Positive solutions for the p-Laplacian: application of the fibering method*, Proc. Royal Soc. Edinburgh Sect A, **127** (1997) 703–726.
[13] A. GHANMI, K. SAOUDI, *A multiplicity results for a singular problem involving the fractional $p-$Laplacian operator*, Complex variables and elliptic equations **61**, 9 (2016) 1199–1216.

Paper FDC-06-13

[14] J. GIACOMONI AND K. SAOUDI, *Multiplicity of positive solutions for a singular and critical problem*, Nonlinear Anal. **71** (2009), no. 9, 4060–4077.
[15] J. GIACOMONI, I. SCHINDLER AND P. TAKÁČ, *Sobolev versus Hölder local minimizers and global multiplicity for a singular and quasilinear equation*, annali della scuola normale superiore di pisa, classe di scienze série V, **6** (2007) 117–158.
[16] E. DI. NEZZA, G. PALATUCCI AND E. VALDINOCI, *Hitchhiker's guide to the fractional Sobolev spaces*, Bull. Sci. Math. **136** (2012) 521–573.
[17] K. SAOUDI AND M. KRATOU, *Existence of multiple solutions for a singular and quasilinear equation*, Complex Var. Elliptic Equ. **60** (2015), no. 7, 893–925.
[18] R. SERVADEI, E. VALDINOCI, *Mountain Pass solutions for non-local elliptic operators*, J. Math. Anal. Appl. **389** (2012), 887–898.
[19] R. SERVADEI, E. VALDINOCI, *Variational methods for non-local operators of elliptic type*, Discrete Contin. Dyn. Syst. **33** (2013), 2105–2137.
[20] R. SERVADEI AND E. VALDINOCI, *On the spectrum of two different fractional operators*, Proc. Roy. Soc. Edinburgh Sect. A **144** (2014), no. 4, 831–855.
[21] J. TAN, *The Brézis-Nirenberg type problem involving the square root of the Laplacian*, Calc. Var. Partial Differential Equations **36** (2011), 21–41.