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# ELASTICITY FOR ECONOMIC PROCESSES WITH MEMORY: FRACTIONAL DIFFERENTIAL CALCULUS APPROACH

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*Abstract.* Derivatives of non-integer orders are applied to generalize notion of elasticity in framework of economic dynamics with memory. Elasticity of $Y$ with respect to $X$ is defined for the case of a finite-interval fading memory of changes of $X$ and $Y$. We define generalizations of point price elasticity of demand to the case of processes with memory. In these generalizations we take into account dependence of demand not only from current price (price at current time), but also all changes of prices for some time interval. For simplification, we will assume that there is one parameter, which characterizes a degree of damping memory over time. The properties of the suggested fractional elasticities and examples of calculations of these elasticities of demand are suggested.

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