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TWO–WEIGHTED INEQUALITY FOR (p,q)–ADMISSIBLE $B_{k,n}$–POTENTIAL OPERATORS IN WEIGHTED LEBESGUE SPACES

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Abstract. In this paper, we study the boundedness of (p,q)-admissible potential operators, associated with the Laplace-Bessel differential operator $B_{k,n}=\sum\limits_{i=1}^{n}\frac{\partial^2}{\partial x_i^2}+\sum\limits_{j=1}^{k}\frac{\gamma_j}{x_j}\frac{\partial}{\partial x_j}$ ((p,q)-admissible $B_{k,n}$-potential operators) on a weighted Lebesgue spaces $L_{p,\omega,\gamma}(\mathbb{R}^n_{k,+})$ including their weak versions. These conditions are satisfied by most of the operators in harmonic analysis, such as the $B_{k,n}$-fractional maximal operator, $B_{k,n}$-potential integral operators and so on. Sufficient conditions on weighted functions ω and ω_1 are given so that (p,q)-admissible $B_{k,n}$-potential operators are bounded from $L_{p,\omega,\gamma}(\mathbb{R}^n_{k,+})$ to $L_{q,\omega_1,\gamma}(\mathbb{R}^n_{k,+})$ for $1<p<q<\infty$ and weak (p,q)-admissible $B_{k,n}$-potential operators are bounded from $L_{p,\omega,\gamma}(\mathbb{R}^n_{k,+})$ to $WL_{q,\omega_1,\gamma}(\mathbb{R}^n_{k,+})$ for $1\leqslant p<q<\infty$.

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