

EXISTENCE OF POSITIVE SOLUTIONS FOR COUPLED SYSTEMS OF HALF-LINEAR BOUNDARY VALUE PROBLEMS INVOLVING CAPUTO FRACTIONAL DERIVATIVES

YOUSUF GHOLAMI AND KAZEM GHANBARI

Abstract. In this paper, we study coupled systems of the half-linear boundary value problems involving left sided Caputo fractional derivatives. The main goal of this paper is restricted to the existence verification of positive solutions for mentioned fractional boundary value problems. To this aim we use nonlinear alternative of Leray-Schauder and Krasnoselskii-Zabreiko fixed point theorems. At the end we present some numerical examples to illustrate the obtained theoretical results.

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