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A FRACTIONAL RATE MODEL OF LEARNING

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Abstract. A fundamental principle of Cognitive Psychology states that the rate at which the human brain learns a certain amount of knowledge is proportional to the amount of knowledge yet to be learned. This is the so called pure memory or tabula raza law of learning. The mathematical formulation of this principle leads to a simple ordinary differential equation of the first order. Here we expand the existing mathematical model to a fractional differential equation which allows for a more realistic model having a much higher freedom to fit possible experimental data, as well as allowing for memory effects during the learning process. Two different definitions of the fractional derivative are used, one is the standard Riemann-Liouville global definition and the other is a local definition based on the choice of the unit that measures functional variation. A detailed comparison with the conventional model both at the analytical and the numerical level is included.

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