

ANALYTIC SOLUTION OF GENERALIZED SPACE TIME FRACTIONAL REACTION DIFFUSION EQUATION

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Abstract. The aim of this paper is to investigate the solution of a generalized space-time fractional reaction-diffusion equation associated with the Hilfer-Prabhakar time fractional derivative and the space fractional Laplacian operator. The solution of the equation in terms of the three parameter Mittag-Leffler function, is obtained by applying the Laplace and Fourier transforms. The work by K. B. Kachhia and Prajapati (2015), R. Garra *et al.* (2014) and S. D. Purohit (2011) and references therein follow as particular cases of our results.

Mathematics subject classification (2010): 35R11, 35C15, 33E12, 26A33.

Keywords and phrases: Linear space time reaction diffusion equation, Fourier transform, Laplace transform, Mittag-Leffler function, composite fractional derivative, fractional Laplace operator.

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