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POSITIVE SOLUTIONS FOR BOUNDARY VALUE PROBLEMS OF N–DIMENSION NONLINEAR FRACTIONAL DIFFERENTIAL SYSTEM WITH INTEGRAL BOUNDARY CONDITIONS

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Abstract. In this paper, we study existence of positive solutions to the system of three-point fractional boundary value problem

$$\begin{cases} D_{0^+}^{\alpha_i} u(t) + \lambda_i a_i(t) f_i(u_1(t), \dots, u_n(t)) = 0, \quad 0 < t < 1, \quad 2 < \alpha_i \leqslant 3 \\ u_i(0) = u_i'(0) = 0 \\ u_i'(1) - \mu_i u_i'(\eta_i) = \int_0^1 \phi_i(s) u_i'(s) ds \end{cases}$$

where for $i = 1, \dots, n$, λ_i is a positive parameter, $D_{0^+}^{\alpha_i}$ is the standard Riemann-Liouville differential operator of order $\alpha_i \in (2,3]$, $\eta_i \in (0,1)$, $\mu_i \geqslant 0$, $f_i : [0,+\infty)^n \to [0,+\infty)$ is a continuous function and $\phi_i : (0,1) \to (0,+\infty)$ is a continuous increasing function and $\int_0^1 s^{\alpha_i - 2} \phi_i(s) ds < +\infty$. Existence results are obtained by means of Krasnosel'skii 's fixed point theorem and the vector version of Krasnosel'skii's fixed point theorem.

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