

EXISTENCE AND UNIQUENESS OF SOLUTIONS FOR NONLINEAR FRACTIONAL NABLA DIFFERENCE SYSTEMS WITH INITIAL CONDITIONS

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Abstract. In this paper, we give sufficient conditions to guarantee the global existence and the uniqueness of solutions of nonlinear fractional nabla difference systems and study the dependence of solutions on initial conditions and parameters.

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