

POSITIVE SOLUTIONS OF A SYSTEM OF FRACTIONAL FUNCTIONAL DIFFERENTIAL EQUATIONS WITH NONLOCAL BOUNDARY CONDITIONS

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Abstract. We study the system of two fractional functional differential equations with the Caputo fractional derivative. Using the Guo–Krasnoselskii fixed point theorem on cones and the nonlinear Leray–Schauder alternative the existence of positive solutions to the system satisfying nonlocal boundary conditions is proved. The boundary conditions are given by linear bounded functionals. Examples are given to illustrate our results.

Mathematics subject classification (2010): 34A08, 26A33, 33E12, 34B18.

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