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KALLMAN–ROTA TYPE INEQUALITY FOR DISCRETE EVOLUTION FAMILIES OF BOUNDED LINEAR OPERATORS

AKBAR ZADA AND USMAN RIAZ

Abstract. Let $\mathscr{X}$ be a complex Banach space and $\mathscr{Z}_+$ be the set of all nonnegative integers. Let $\mathscr{K}_{00}(\mathscr{Z}_+,\mathscr{X})$ be the space of all $\mathscr{X}$-valued bounded sequences which decays to zero at 0 and at ∞. Using the space $\mathscr{K}_{00}(\mathscr{Z}_+,\mathscr{X})$, we give Kallman-Rota type inequality for the discrete evolution family $\mathscr{U}=\{U(m,n):m,n\in\mathscr{Z}_+,m\geqslant n\}$ of bounded linear operators. We also present the same inequality for (r,q)-resolvent operators, which arises in the solution of fractional difference equation. In particular, if $\mathscr{A}$ is the algebraic generator of α-times family of bounded and linear operators, arising from the well posedness of fractional difference equations of order $\beta+1$, then we prove that the inequality

$$\|\mathscr{A}x\|^2\leqslant 8\eta^2\frac{\Gamma(\alpha+\beta+2)^2}{\Gamma(\alpha+1)\Gamma(\alpha+2\beta+3)}\|x\|\|\mathscr{A}^2x\|,$$

holds for all $x\in D(\mathscr{A}^2)$.

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