

## FORCED OSCILLATION OF FRACTIONAL PARTIAL DIFFERENTIAL EQUATIONS WITH DAMPING TERM

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*Abstract.* In this paper, the forced oscillation of fractional partial differential equations of the form

$$D_{+,t}^{1+\alpha}u(x,t) + p(t)D_{+,t}^{\alpha}u(x,t) = a(t)\Delta u(x,t) + \sum_{i=1}^m a_i(t)\Delta u(x,t - \tau_i) - q(x,t) \int_0^t (t - \xi)^{-\alpha} u(x, \xi) d\xi + f(x,t), \quad (x,t) \in \Omega \times \mathbb{R}_+ \equiv G$$

are investigated, and some examples are given to illustrate the usefulness of our results.

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