

## QUASI-PERIODIC SOLUTIONS OF FRACTIONAL NABLA DIFFERENCE SYSTEMS

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*Abstract.* In this paper, we analyse periodic properties of fractional nabla difference systems. First, we prove that a system of fractional nabla difference equations with a periodic right hand side can not possess a periodic solution. Next, we establish sufficient conditions on the existence of a unique  $S$ -asymptotically  $T$ -periodic solution for this difference system. Finally, we provide an example illustrating the obtained results.

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