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SHIFTED CHEBYSHEV SPECTRAL–COLLOCATION METHOD FOR SOLVING OPTIMAL CONTROL OF FRACTIONAL MULTI–STRAIN TUBERCULOSIS MODEL

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Abstract. In this paper, optimal control for a novel fractional multi-strain Tuberculosis model is presented. The proposed model is governed by a system of fractional differential equations, where the fractional derivative is defined in the Caputo sense. Modified parameters are introduced to account for the fractional order. Four controls variables are proposed to minimize the cost of interventions. Necessary and sufficient conditions that guarantee the existence and the uniqueness of the solution of the resulting systems are given. The optimality system is approximated by shifted Chebyshev polynomials which transformed the system of differential equations to a nonlinear system of algebraic equations with unknown coefficients. The convergence analysis and an upper bound of the error of the derived formula are given. Newton's iteration method is used to solve this system of nonlinear algebraic equations. The value of the objective functional which is obtained by the proposed method are compared with those obtained by the generalized Euler method. It is found that, Shifted Chebyshev spectral-collocation method is better than the generalized Euler method.

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