

LYAPUNOV INEQUALITIES FOR TWO KINDS OF HIGHER-ORDER MULTI-POINT FRACTIONAL BOUNDARY VALUE PROBLEMS

DEXIANG MA

Abstract. We obtain Lyapunov inequality and Hartman-Wintner-type inequality for two higher-order multi-point fractional boundary value problems. The technique of order-reduction and the properties of Green's function are important to our results. As applications, we discuss the eigenvalue problem and the real zeros for Mittag-Leffler function.

Mathematics subject classification (2010): 26A33, 26D10, 33E12, 34A08.

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