LYAPUNOV INEQUALITIES FOR TWO KINDS OF HIGHER–ORDER MULTI–POINT FRACTIONAL BOUNDARY VALUE PROBLEMS

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Abstract. We obtain Lyapunov inequality and Hartman-Wintner-type inequality for two higher-order multi-point fractional boundary value problems. The technique of order-reduction and the properties of Green's function are important to our results. As applications, we discuss the eigenvalue problem and the real zeros for Mittag-Leffler function.

Mathematics subject classification (2010): 26A33, 26D10, 33E12 , 34A08.
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REFERENCES

[1] A. M. LYAPUNOV, *Probléme général de la stabilité du mouvement*, Annales de la Faculté des sciences de Toulouse: Mathématiques, **9** (1907), 203–474.
[2] A. WINTNER, *On the non-existence of conjugate points*, Amer. J. Math. **73** (1951), 368–380.
[3] P. HARTMAN AND A. WINTNER, *On an oscillation criterion of Lyapunov* Amer. J. Math. **73** (1951), 885–890.
[4] A. TIRYAKI, *Recent developments of Lyapunov-type inequalities*, Adv. Dyn. Syst. Appl. **5**: 2 (2010), 231–248.
[5] R. P. AGARWAL AND A. ÖZBEKLER, *Lyapunov type inequalities for second order forced mixed nonlinear impulsive differential equations*, Appl. Math. Comput. **282** (2016), 216–225.
[6] R. P. AGARWAL AND A. ÖZBEKLER, *Lyapunov type inequalities for second order sub and super-half-linear differential equations*, Dynam. Syst. Appl. **24** (2015), 211–220.
[7] R. P. AGARWAL AND A. ÖZBEKLER, *Lyapunov type inequalities for even order differential equations with mixed nonlinearities*, J. Inequal. Appl. **2015**: 142 (2015), 1–10.
[8] R. A. C. FERREIRA, *A Lyapunov-type inequality for a fractional boundary value problem*, Fract. Calc. Appl. Anal. **16**: 4 (2013), 978–984.
[9] R. A. C. FERREIRA, *On a Lyapunov-type inequality and the zeros of a certain Mittag-Leffler function*, J. Math. Anal. Appl. **412**: 2 (2014), 1058–1063.
[10] M. JLELI AND B. SAMET, *Lyapunov-type inequalities for a fractional differential equation with mixed boundary conditions*, Math. Inequal. Appl. **18**: 2 (2015), 443–451.
[11] M. JLELI, L. RAGOUB AND B. SAMET, *Lyapunov-type inequality for a fractional differential equation under a Robin boundary conditions*, J. Func. Spaces **2015**, Article number 468536 (2015), 1–5.
[12] J. RONG AND C. Z. BAI, *Lyapunov-type inequality for a fractional differential equation with fractional boundary conditions*, Adv. Differ. Equ. **82**: 1 (2015), 1–10.
[13] Y. Y. WANG, S. L. LIANG AND C. X. XIA, *A Lyapunov-type inequalities for a fractional differential equation under sturm-liouville boundary conditions*, Math. Inequal. Appl. **2016**, Article number mia-5033 (2016), 1–10.
[14] R. A. C. FERREIRA, *Lyapunov-type inequalities for some sequential fractional boundary value problems*, Adv. Dyn. Syst. Appl. **11**: 1 (2016), 33–43.
[15] R. P. AGARWAL AND A. ÖZBEKLER, *Lyapunov-type inequalities for mixed nonlinear Riemann-Liouville fractional differential equations with a forcing term*, J. Comput. Appl. Math. (2016), http://dx.doi.org/10.1016/j.cam.2016.10.009.

[16] A. CHIDOUH AND D. F. M. TORRESB, *A generalized Lyapunov inequality for a fractional boundary value problem*, J. Comput. Appl. Math. (2016), DOI.org/10.1016/ j.cam.2016.03.035.

[17] D. O'REGAN AND B. SAMET, *Lyapunov-type inequalities for a class of fractional differential equations*, J. Inequal. Appl. **2015**: 247 (2015), 1–10.

[18] N. ARIFI, I. ALTUN, M. JLELI, A. LASHIN AND B. SAMET, *Lyapunov-type inequalities for a fractional p -Laplacian equation*, J. Inequal. Appl. **2016**: 189 (2016), 1–11.

[19] I. CABRERA, K. SADARANGANI AND B. SAMET, *Hartman-Wintner-type inequalities for a class of nonlocal fractional boundary value problems*, Math. Meth. Appl. Sci. **2016**, DOI: 10.1002/mma.3972 (2016), 1–8.

[20] M. JLELI, M. KIRANE AND B. SAMET, *Lyapunov-type inequalities for fractional partial differential equations*, App. Math. Lett. **66** (2017), 30–39.

[21] M. JLELI, M. KIRANE AND B. SAMET, *Hartman-Wintner-Type inequality for a fractional boundary value problem via a fractional derivative with respect to another function*, Discrete Dyn. Nat. Soc. **2017** (2017).

[22] A. A. KILBAS, H. M. SRIVASTAVA AND J. J. TRUJILLO, *Theory and applications of fractional differential equations*, Amsterdam: Elsevier; 2006.

[23] I. PODLUBNY, *Fractional differential equation*, San Diego, CA: Academic Press; 1999.

[24] A. A. KILBAS, M. RIVERO, L. RODRIGUEZ-GERMA AND J. J. TRUJILLO, *Caputo linear fractional differential equation*, IFAC Proceedings **39**: 11 (2010), 52–57.

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