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ORTHOGONALITY FOR A CLASS OF GENERALISED JACOBI POLYNOMIAL $P_\nu^{\alpha,\beta}(x)$

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Abstract. This work considers g-Jacobi polynomials, a fractional generalisation of the classical Jacobi polynomials. We discuss the polynomials and compare some of their properties to the classical case. The main result of the paper is to show that one can derive an orthogonality property for a sub-class of g-Jacobi polynomials $P_\nu^{(\alpha,\beta)}$. The paper concludes with an application in modelling of ophthalmic surfaces.

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