

EXISTENCE THEORY FOR FRACTIONAL–ORDER NEUTRAL BOUNDARY VALUE PROBLEMS

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Abstract. A new class of Dirichlet boundary value problems of Caputo-Hadamard type fractional neutral differential equations and inclusions is studied. Expressions for Green's functions are derived to obtain the integral equation equivalent to the associated single-valued problem. Existence and uniqueness results are proved for single-valued and multivalued problems at hand. Examples demonstrating the application of the main results are presented. Finally we extend our discussion to the case of three-point nonlocal boundary conditions.

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