

## INEQUALITIES OF JENSEN'S TYPE FOR GENERALIZED $k$ - $g$ -FRACTIONAL INTEGRALS OF FUNCTION $f$ FOR WHICH THE COMPOSITE $f \circ g^{-1}$ IS CONVEX

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*Abstract.* In this paper we establish some inequalities of Jensen and Hermite-Hadamard type for the  $k$ - $g$ -fractional integrals of function  $f$  for which the composite function  $f \circ g^{-1}$  is convex. Some examples for the *generalized left- and right-sided Riemann-Liouville fractional integrals* of a function  $f$  with respect to another function  $g$  on  $[a, b]$  are also given. Applications for Hadamard fractional integrals are provided as well.

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