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INEQUALITIES OF JENSEN'S TYPE FOR GENERALIZED $k-g$–FRACTIONAL INTEGRALS OF FUNCTION f FOR WHICH THE COMPOSITE $f\circ g^{-1}$ IS CONVEX

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Abstract. In this paper we establish some inequalities of Jensen and Hermite-Hadamard type for the k-g-fractional integrals of function f for which the composite function $f\circ g^{-1}$ is convex. Some examples for the *generalized left-* and *right-sided Riemann-Liouville fractional integrals* of a function f with respect to another function g on $[a,b]$ are also given. Applications for Hadamard fractional integrals are provided as well.

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