

BOUNDARY VALUE PROBLEMS FOR HADAMARD FRACTIONAL DIFFERENTIAL EQUATIONS WITH NONLOCAL MULTI-POINT BOUNDARY CONDITIONS

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Abstract. This paper is devoted to the study of the existence and uniqueness of solutions to a boundary value problem for nonlinear Hadamard fractional differential equations with nonlocal multi-point boundary conditions. The results are obtained by using a variety of fixed point theorems. The paper concludes with some illustrative examples.

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