

## A PROBLEM INVOLVING A NONLOCAL OPERATOR

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*Abstract.* The aim of this paper is to deal with the elliptic pdes involving a nonlinear integro-differential operator which are possibly degenerate and covers the case of fractional  $p$ -Laplacian operator. We prove the existence of a solution in the weak sense to the problem

$$\begin{aligned} -\mathcal{L}_{\Phi}u &= \lambda|u|^{q-2}u \text{ in } \Omega, \\ u &= 0 \text{ in } \mathbb{R}^N \setminus \Omega, \end{aligned}$$

if and only if a weak solution to

$$\begin{aligned} -\mathcal{L}_{\Phi}u &= \lambda|u|^{q-2}u + f, \quad f(\neq 0) \in L^{p'}(\Omega), \\ u &= 0 \text{ in } \mathbb{R}^N \setminus \Omega, \end{aligned}$$

( $p'$  being the conjugate of  $p$ ), exists for  $q \in (p, p_s^*)$  under certain condition on  $\lambda$ , where  $-\mathcal{L}_{\Phi}$  is a general nonlocal integro-differential operator of order  $s \in (0, 1)$  and  $p_s^*$  is the fractional Sobolev conjugate of  $p$ . We further prove a necessary condition for the existence of a weak solution to the problem

$$\begin{aligned} -\mathcal{L}_{\Phi}u &= \lambda|u|^{q-2}u + \mu \text{ in } \Omega, \\ u &= 0 \text{ in } \mathbb{R}^N \setminus \Omega, \end{aligned}$$

where  $\mu$  is a measure.

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