

A PROBLEM INVOLVING A NONLOCAL OPERATOR

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Abstract. The aim of this paper is to deal with the elliptic pdes involving a nonlinear integro-differential operator which are possibly degenerate and covers the case of fractional p -Laplacian operator. We prove the existence of a solution in the weak sense to the problem

$$\begin{aligned} -\mathcal{L}_{\Phi}u &= \lambda|u|^{q-2}u \text{ in } \Omega, \\ u &= 0 \text{ in } \mathbb{R}^N \setminus \Omega, \end{aligned}$$

if and only if a weak solution to

$$\begin{aligned} -\mathcal{L}_{\Phi}u &= \lambda|u|^{q-2}u + f, \quad f(\neq 0) \in L^{p'}(\Omega), \\ u &= 0 \text{ in } \mathbb{R}^N \setminus \Omega, \end{aligned}$$

(p' being the conjugate of p), exists for $q \in (p, p_s^*)$ under certain condition on λ , where $-\mathcal{L}_{\Phi}$ is a general nonlocal integro-differential operator of order $s \in (0, 1)$ and p_s^* is the fractional Sobolev conjugate of p . We further prove a necessary condition for the existence of a weak solution to the problem

$$\begin{aligned} -\mathcal{L}_{\Phi}u &= \lambda|u|^{q-2}u + \mu \text{ in } \Omega, \\ u &= 0 \text{ in } \mathbb{R}^N \setminus \Omega, \end{aligned}$$

where μ is a measure.

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