# DETERMINATION OF A DISTRIBUTION IN A SOURCE TERM
# OF A TIME FRACTIONAL DIFFUSION–WAVE EQUATION

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*Abstract.* We study the inverse Cauchy problem to a time fractional diffusion-wave equation with distributions in right-hand sides. This problem is to find a generalized solution of direct problem and an unknown time-dependent part of a source from the space of distributions. The unique solvability of the problem is established.

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