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EXISTENCE OF SOLUTIONS FOR A CLASS OF FRACTIONAL HAMILTONIAN SYSTEMS WITH IMPULSIVE EFFECTS

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Abstract. In this paper, we are concerned with a class of fractional Hamiltonian systems containing right Riemann-Liouville fractional derivatives and left Caputo fractional derivatives with impulsive effects. Under certain conditions, the existence of solutions are obtained for this class of systems by means of the least action principle, the saddle point theorem as well as some skills of inequalities. One of the innovations of this paper is that the variational functional of these problems are established in a proper fractional derivative space. Moreover, in order to show the feasibility and effectiveness of our results, we present two examples.

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