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GENERALIZED ALMOST AUTOMORPHIC AND GENERALIZED ASYMPTOTICALLY ALMOST AUTOMORPHIC SOLUTIONS OF ABSTRACT VOLTERRA INTEGRO–DIFFERENTIAL INCLUSIONS

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Abstract. The main aim of this paper is to investigate generalized almost automorphy and generalized asymptotical almost automorphy of solutions for certain classes of abstract Volterra integro-differential inclusions and abstract (semilinear) fractional differential inclusions in Banach spaces. We illustrate our abstract results with several examples and possible applications.

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REFERENCES

[1] S. ABBAS, *A note on Weyl pseudo almost automorphic functions and their properties*, Math. Sci. (Springer), **6:29** (2012), 5 pp, doi:10.1186/2251-7456-6-29.
[2] S. ABBAS, V. KAVITHA, AND R. MURUGESU, *Stepanov-like weighted pseudo almost automorphic solutions to fractional order abstract integro-differential equations*, Proc. Indian Acad. Sci. (Math. Sci.), **125** (2015), 323–351.
[3] M. AMERIO AND G. PROUSE, *Almost Periodic Functions and Functional Equations*, Van Nostrand-Reinhold, New York, 1971.
[4] J. ANDRES, A. M. BERSANI, AND R. F. GRANDE, *Hierarchy of almost-periodic function spaces*, Rend. Mat. Appl. **26** (2006), 121–188.
[5] W. ARENDT, C. J. K. BATTY, M. HIEBER, AND F. NEUBRANDER, *Vector-valued Laplace Transforms and Cauchy Problems*, Birkhäuser/Springer Basel AG, Basel, 2001.
[6] E. BAZHLEKOVA, *Fractional evolution equations in Banach spaces*, Ph. D. thesis, Eindhoven University of Technology, Eindhoven, 2001.
[7] F. BEDOUHENE, N. CHALLALI, O. MELLAH, P. RAYNAUD DE FITTE, AND M. SMAALI, *Almost automorphy and various extensions for stochastic processes*, J. Math. Anal. Appl. **429** (2015), 1113–1152.
[8] A. S. BESICOVITCH, *Almost Periodic Functions*, Dover Publications Inc., New York, 1954.
[9] J. BLOT, G. M. MOPHOU, G. M. N'GUÉRÉKATA, AND D. PENNEQUIN, *Weighted pseudo almost automorphic functions and applications to abstract differential equations*, Nonlinear Anal. **71** (2009), 903–909.
[10] S. BOCHNER, *A new approach to almost periodicity*, Proc. Nat. Acad. Sci. USA., **48** (1962), 2039–2043.
[11] D. BUGAJEWSKI AND T. DIAGANA, *Almost automorphy of the convolution operator and applications to differential and functional differential equations*, Nonlinear Stud. **13** (2006), 129–140.
[12] R. W. CARROLL AND R. W. SHOWALTER, *Singular and Degenerate Cauchy Problems*, Academic Press, New York, 1976.
[13] D. N. CHEBAN, *Asymptotically Almost Periodic Solutions of Differential Equations*, Hindawi Publishing Corporation, New York, 2009.

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[14] V. CASARINO, *Almost automorphic groups and semigroups*, Rend. Accad. Naz. Sci. XL Mem. Mat. Appl. (5), **24** (2000), 219–235.

[15] R. CROSS, *Multivalued Linear Operators*, Marcel Dekker Inc., New York, 1998.

[16] C. CUEVAS AND C. LIZAMA, *Almost automorphic solutions to a class of semilinear fractional differential equations*, Appl. Math. Lett. **21** (2008), 1315–1319.

[17] T. DIAGANA, *Almost Automorphic Type and Almost Periodic Type Functions in Abstract Spaces*, Springer, New York, 2013.

[18] T. DIAGANA, G. M. N'GUÉRÉKATA, AND N. V. MINH, *Almost automorphic solutions to evolution equations*, Proc. Amer. Math. Soc. **132** (2004), 3289–3298.

[19] T. DIAGANA AND G. M. N'GUÉRÉKATA, *Almost automorphic solutions to semilinear evolution equations*, Funct. Differ. Equ. **13** (2006), 195–206.

[20] T. DIAGANA AND R. AGARWAL, *Existence of pseudo almost automorphic solutions for the heat equation with S^p -pseudo almost automorphic coefficients*, Boundary Value Problems, vol. 2009, Article ID 182527, 19 pages, doi:10.1155/2009/182527.

[21] T. DIAGANA, V. NELSON, AND G. M. N'GUÉRÉKATA, *Stepanov-like $C^{(n)}$ -pseudo-almost automorphy and applications to some nonautonomous higher-order differential equations*, Opuscula Math. **32** (2012), 455–471.

[22] H.-S. DING, J. LIANG, AND T.-J. XIAO, *Almost automorphic solutions to nonautonomous semilinear evolution equations in Banach spaces*, Nonlinear Anal. **73** (2010), 1426–1438.

[23] H.-S. DING, J. LIANG, AND T.-J. XIAO, *Some properties of Stepanov-like almost automorphic functions and applications to abstract evolution equations*, Appl. Anal. **88** (2009), 1079–1091.

[24] H.-S. DING, J. LIANG, AND T.-J. XIAO, *Almost automorphic solutions to abstract fractional differential equations*, Advances Diff. Equ., vol. 2010, Article ID 508374, 9 pages, doi:10.1155/2010/508374.

[25] Z. FAN, J. LIANG, AND T.-J. XIAO, *On Stepanov-like (pseudo)-almost automorphic functions*, Nonlinear Anal. **74** (2011), 2853–2861.

[26] S. FATAJOU, N. VAN MINH, G. M. N'GUÉRÉKATA, AND A. PANKOV, *Stepanov-like almost automorphic solutions for nonautonomous evolution equations*, Electron. J. Differential Equations **121** (2007), 1–11.

[27] A. FAVINI AND A. YAGI, *Degenerate Differential Equations in Banach Spaces*, Chapman and Hall/CRC Pure and Applied Mathematics, New York, 1998.

[28] J. A. GOLDSTEIN AND G. M. N'GUÉRÉKATA, *Almost automorphic solutions of semilinear evolution equations*, Proc. Amer. Math. Soc. **133** (2005), 2401–2408.

[29] G. M. N'GUÉRÉKATA, *Almost Automorphic and Almost Periodic Functions in Abstract Spaces*, Kluwer Acad. Publ., Dordrecht, 2001.

[30] G. M. N'GUÉRÉKATA AND M. KOSTIĆ, *Generalized asymptotically almost periodic and generalized asymptotically almost automorphic solutions of abstract multi-term fractional differential inclusions*, Abstract Appl. Anal., volume 2018, Article ID 5947393, 17 pages, https://doi.org/10.1155/2018/5947393.

[31] Y. HINO, T. NAITO, N. V. MINH, AND J. S. SHIN, *Almost Periodic Solutions of Differential Equations in Banach Spaces*, Stability and Control: Theory, Methods and Applications **15**, Taylor and Francis Group, London, 2002.

[32] M. KOSTIĆ, *Abstract Volterra Integro-Differential Equations*, Taylor and Francis Group/CRC Press/Science Publishers, Boca Raton, New York, 2015.

[33] M. KOSTIĆ, *Abstract Degenerate Volterra Integro-Differential Equations: Linear Theory and Applications*, Book Manuscript, 2016.

[34] M. KOSTIĆ, *On almost periodic solutions of abstract semilinear fractional inclusions with Weyl-Liouville derivatives of order $\gamma \in (0,1]$*, J. Math. Stat. **13** (2017), 240–250.

[35] M. KOSTIĆ, *The existence and uniqueness of pseudo-almost periodic solutions of semilinear Cauchy inclusions of first order*, Appl. Math. Comp. Sci. **2** (2017), 19–24.

[36] M. KOSTIĆ, *Abstract degenerate fractional differential inclusions*, Appl. Anal. Discrete Math. **11** (2017), 39–61.

[37] M. KOSTIĆ, *Existence of generalized almost periodic and asymptotic almost periodic solutions to abstract Volterra integro-differential equations*, Electron. J. Differential Equations, vol. 2017, no. **239** (2017), 1–30.

[38] M. KOSTIĆ, *Almost Periodic and Almost Automorphic Type Solutions of Abstract Volterra Integro-Differential Equations*, Book Manuscript, 2017.
[39] M. KOSTIĆ, *Weyl-almost periodic solutions and asymptotically Weyl-almost periodic solutions of abstract Volterra integro-differential equations*, J. Math. Anal. Appl., submitted.
[40] M. KOSTIĆ, *The existence and uniqueness of almost periodic and asymptotically almost periodic solutions of semilinear Cauchy inclusions*, Hacetepe J. Math. Stat., submitted.
[41] H. LEE AND H. ALKAHBY, *Stepanov-like almost automorphic solutions of nonautonomous semilinear evolution equations with delay*, Nonlinear Anal. **69** (2008), 2158–2166.
[42] M. LEVITAN AND V. V. ZHIKOV, *Almost Periodic Functions and Differential Equations*, Cambridge Univ. Press, London, 1982.
[43] J. LIANG, J. ZHANG, AND T.-J. XIAO, *Composition of pseudo-almost automorphic and asymptotically almost automorphic functions*, J. Math. Anal. Appl. **340** (2008), 1493–1499.
[44] I. V. MELNIKOVA AND A. I. FILINKOV, *Abstract Cauchy Problems: Three Approaches*, Chapman Hall/CRC Press, Boca Raton, 2001.
[45] J. MU, Y. ZHOA, AND L. PENG, *Periodic solutions and S-asymptotically periodic solutions to fractional evolution equations*, Discrete Dyn. Nat. Soc. vol. 2017, Article ID 1364532, 12 pages, https://doi.org/10.1155/2017/1364532.
[46] F. PERIAGO AND B. STRAUB, *A functional calculus for almost sectorial operators and applications to abstract evolution equations*, J. Evol. Equ. **2** (2002), 41–68.
[47] J. PRÜSS, *Evolutionary Integral Equations and Applications*, Birkhäuser-Verlag, Basel, 1993.
[48] A. REICH, *Präkompakte Gruppen und Fastperiodizität*, Math. Z. **116** (1970).
[49] S. G. SAMKO, A. A. KILBAS, AND O. I. MARICHEV, *Fractional Derivatives and Integrals: Theory and Applications*, Gordon and Breach, New York, 1993.
[50] G. A. SVIRIDYUK AND V. E. FEDOROV, *Linear Sobolev Type Equations and Degenerate Semigroups of Operators*, Inverse and Ill-Posed Problems (Book 42), VSP, Utrecht, Boston, 2003.
[51] R. TERRAS, *Almost automorphic functions on topological groups*, Indiana U. Math., J. **21** (1972).
[52] W. A. VEECH, *Almost automorphic functions on groups*, Amer. J. Math. **87** (1965), 719–751.
[53] W. A. VEECH, *On a theorem of Bochner*, Ann. of Math. **86** (1967), 117–137.
[54] W. VON WAHL, *Gebrochene Potenzen eines elliptischen Operators und parabolische Differentialgleichungen in Räumen hölderstetiger Funktionen*, Nachr. Akad. Wiss. Göttingen Math.-Phys. Kl. **11** (1972), 231–258.
[55] R.-N. WANG, D.-H. CHEN, T.-J. XIAO, *Abstract fractional Cauchy problems with almost sectorial operators*, J. Differential Equations **252** (2012), 202–235.
[56] Z. XIA AND M. FAN, *Weighted Stepanov-like pseudo almost automorphy and applications*, Nonlinear Anal. **75** (2012), 2378–2397.
[57] T.-J. XIAO, J. LIANG, AND J. ZHANG, *Pseudo-almost automorphic solutions to semilinear differential equations in Banach spaces*, Semigroup Forum **76** (2006), 518–524.
[58] S. ZAIDMAN, *Almost-Periodic Functions in Abstract Spaces*, Pitman Research Notes in Math., vol. **126**, Pitman, Boston, 1985.
[59] M. ZAKI, *Almost automorphic solutions of certain abstract differential equations*, Ann. Mat. Pura Appl. **101** (1974).

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