NEW ϕ –FRACTIONAL ESTIMATES OF
HERMITE–HADAMARD TYPE INEQUALITIES

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Abstract. Using the approach of ϕ -fractional calculus a new estimate of Hermite-Hadamard type inequality via h -preinvex functions is established. Two new ϕ -fractional integral identities are obtained. With the help of these identities some more integral inequalities of Hermite-Hadamard type are derived via twice differentiable s -Breckner and s -Godunova-Levin-Dragomir preinvex functions. Some special cases for the results obtained in the paper are also discussed.

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