

## MINIMAL AND MAXIMAL SOLUTIONS FOR A FRACTIONAL BOUNDARY VALUE PROBLEM AT RESONANCE ON THE HALF LINE

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*Abstract.* This paper is devoted to the study of a Riemann-Liouville fractional boundary value problem on an unbounded interval. The problem is assumed to be at resonance and the boundary conditions are of nonlocal type. We obtain some existence results for the maximal and minimal solutions by means of a fixed point theorem for an increasing operator and lower and upper solutions.

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