SOME NEW HERMITE–HADAMARD TYPE INEQUALITIES VIA CAPUTO k–FRACTIONAL DERIVATIVES CONCERNING $(n+1)$–DIFFERENTIABLE GENERALIZED RELATIVE SEMI–$(r;m,h_1,h_2)$–PREINVEX MAPPINGS

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Abstract. In this article, we first presented a new identity concerning $(n+1)$-differentiable mappings defined on m-invex set via Caputo k-fractional derivatives. By using the notion of generalized relative semi-$(r;m,h_1,h_2)$-preinvexity and the obtained identity as an auxiliary result, some new estimates with respect to Hermite-Hadamard type inequalities via Caputo k-fractional derivatives are established. It is pointed out that some new special cases can be deduced from main results of the article.

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