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HILFER AND HADAMARD COUPLED VOLTERRA FRACTIONAL INTEGRO–DIFFERENTIAL SYSTEMS WITH RANDOM EFFECTS

SAÏD ABBAS, RAVI P. AGARWAL, MOUFFAK BENCHOHRA AND BOUALEM ATTOU SLIMANI

Abstract. This paper deals with some existence results for two classes of coupled systems of Hilfer and Hilfer-Hadamard random fractional integro-differential equations. The main tool used to carry out our results is Itoh's random fixed point theorem.

Mathematics subject classification (2010): 26A33.

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