

ON THE WEAK SOLUTION $u \in C_{1-\alpha}(I, E)$ OF A FRACTIONAL-ORDER WEIGHTED CAUCHY TYPE PROBLEM IN REFLEXIVE BANACH SPACES

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Abstract. In this paper, we study the existence of a weak solution $u \in C_{1-\alpha}(I, E)$ of the nonlinear weighted Cauchy type problem of fractional-order.

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