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SOME NEW HERMITE–HADAMARD TYPE INEQUALITIES VIA k–FRACTIONAL INTEGRALS CONCERNING DIFFERENTIABLE GENERALIZED–$\mathbf{m}$–$((h_1^p,h_2^q);(\eta_1,\eta_2))$–CONVEX MAPPINGS

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Abstract. The authors discovered a new identity concerning differentiable mappings defined on $\mathbf{m}$-invex set via k-fractional integrals. By using the obtained identity as an auxiliary result, some new estimates with respect to Hermite-Hadamard type inequalities via k-fractional integrals for generalized-$\mathbf{m}$-$((h_1^p,h_2^q);(\eta_1,\eta_2))$-convex mappings are presented. It is pointed out that some new special cases can be deduced from main results. At the end, some applications to special means for different positive real numbers are provided as well.

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