

GLOBAL STABILITY OF A LESLIE–GOWER–TYPE FRACTIONAL ORDER TRITROPHIC FOOD CHAIN MODEL

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Abstract. Recently, the dynamical behaviors of a fractional order three species food chain model was studied by Alidousti and Ghahfarokhi (*Nonlinear Dynamics*, doi: [org/10.1007/s11071-018-4663-6](https://doi.org/10.1007/s11071-018-4663-6), 2018). They proved both the local and global asymptotic stability of all equilibrium points except the interior one. This work extends their work and gives proof of both the local and global stability analysis of the interior equilibrium point. Numerical examples are also provided to substantiate the analytical findings.

Mathematics subject classification (2010): 34A08, 26A33, 34K37, 44A99.

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