SOME INEQUALITIES FOR THE GENERALIZED
$k−g−$FRACTIONAL INTEGRALS OF CONVEX FUNCTIONS

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Abstract. Let g be a strictly increasing function on (a,b), having a continuous derivative g' on (a,b). For the Lebesgue integrable function $f : (a,b) \to \mathbb{C}$, we define the k-g-*left-sided fractional integral* of f by

$$S_{k,g,a+} f(x) = \int_a^x k(g(x) - g(t)) g'(t) f(t) \, dt, \; x \in (a,b]$$

and the k-g-*right-sided fractional integral* of f by

$$S_{k,g,b-} f(x) = \int_x^b k(g(t) - g(x)) g'(t) f(t) \, dt, \; x \in [a,b),$$

where the kernel k is defined either on $(0,\infty)$ or on $[0,\infty)$ with complex values and integrable on any finite subinterval.

In this paper we establish some trapezoid and Ostrowski type inequalities for the k-g-fractional integrals of convex functions. Applications for Hermite-Hadamard type inequalities for generalized g-means and examples for Riemann-Liouville and exponential fractional integrals are also given.

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